

## UNIT-5

### CONSOLIDATION

#### INTRODUCTION :-

The property of soil mass at which the change in volume or decrease in volume takes place under compressive forces is known as compressibility of soil.

compression of solid particles and water in the voids

- (a) compression of solid particles and water in the voids
- (b) Compression and expulsion of air in the voids
- (c) Expulsion of water in voids.

Settlement :- plastic readjustment of solid particles with the expulsion of air by , compaction, consolidation.

Expulsion of air + water + adsorbed water + plastic readjustment of solid particles (Settlement)

#### Stages of Consolidation :-

consolidation of soil deposit can be divided into 3 stages

- ① Initial consolidation
- ② Primary consolidation
- ③ Secondary Consolidation.

① Initial consolidation :- When load is applied to a partially saturated soils, decrease in volume occur due to expulsion of air in the voids

\* The Reduction in the volume of the soil just after the application of the load is known as "initial consolidation" (or) "initial compression".

② primary consolidation :- After initial consolidation, further reduction occurs due to expulsion of water from voids

\* The Reduction in volume due to expulsion of water is known as "primary consolidation".

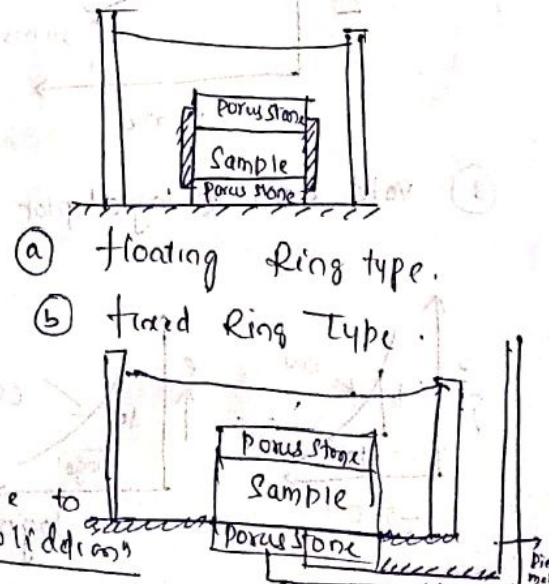
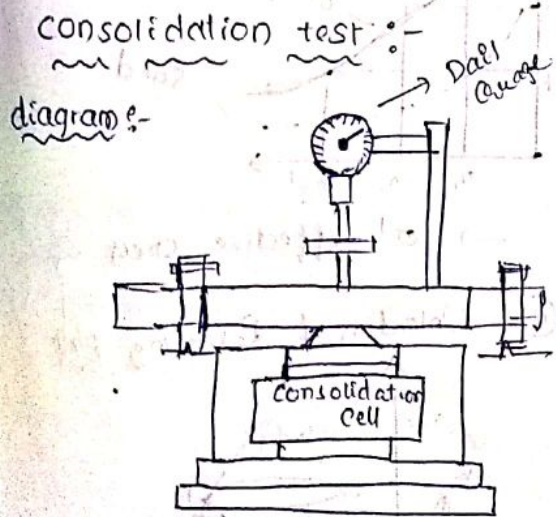
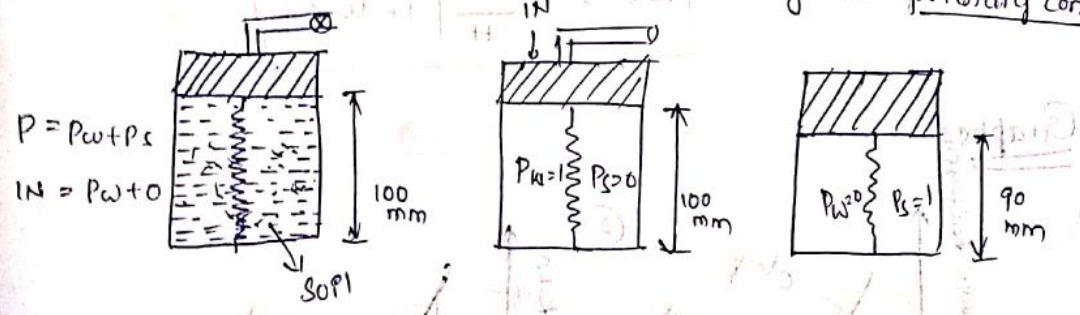
- \* the decrease depends upon the permeability of soil and its time dependent.
- \* the fine grained soils, the primary consolidation occurs over a long time.
- \* In coarse grained soils, the primary consolidation occurs rather than quickly due to high permeability.

② Secondary Consolidation :- The additional reduction in volume of soil at a slow rate is called as "Secondary consolidation"

- \* the decrease in the volume of soil mass due to adsorbed water and also plastic readjustment of solid particles is called as "Secondary consolidation"
- \* Secondary consolidation is important for certain types of soil, such as peats and soft organic clays.

Consolidation mean primary consolidation unless otherwise stated. It is the most important component of the total consolidation.

Spring Analogy :- It is used for determining the "primary consolidation"



The decrease in the vol of soil due to expulsion of water is called "consolidation"



Applied Pressure | Dial gauge Reading |  $\Delta H$  change in thickness (or) height

① Height of solids method.

(partially saturated + fully saturated)  
we can use this method

$$e = \frac{V_v}{V_s} = \frac{V - V_s}{V_s}$$

$$e = \frac{A \times H - A \times H_s}{A \times H_s} = \left[ e = \frac{H - H_s}{H_s} \right]$$

$$G = \frac{W_s}{W} \Rightarrow W_s = G W \Rightarrow \frac{W_s}{V_s} = G \gamma_w$$

$$V_s = \frac{W_s}{G \gamma_w} \Rightarrow A \times H_s = \frac{W_s}{G \gamma_w}$$

$$H_s = \frac{W_s}{G A \gamma_w}$$

② change in void ratio method.

$$\frac{\Delta H}{H} = \frac{\Delta e}{1+e} \quad (w.k.th \ e_s = W G)$$

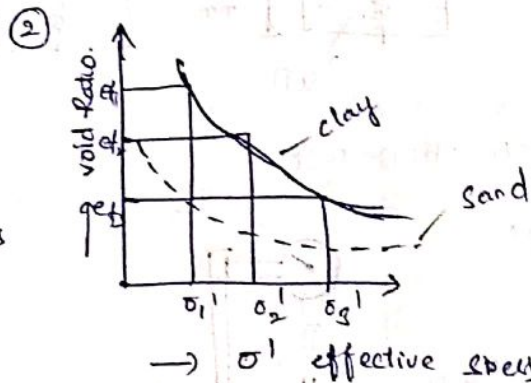
$S=1$  for fully saturated soils

$$\Delta e = \frac{\Delta H}{H} [1+e]$$

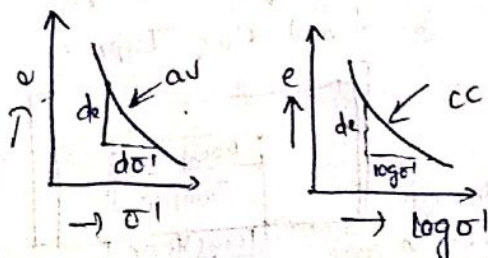
$$e = W G$$

$$\Delta e = \frac{\Delta H}{H} [1 + W G]$$

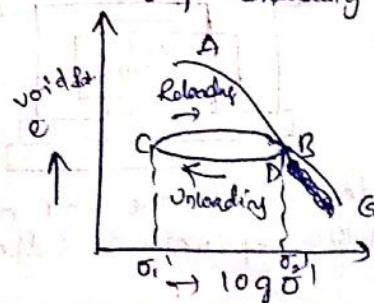
### Graphs



③ void ratio  $v$  vs  $\log \sigma'$  plot



④ Unloading & Reloading plot



Basic Definitions :-

co-efficient of compressibility ( $a_v$ ) :-

$$a_v = - \frac{de}{d\sigma'}$$

$$\therefore a_v = \frac{e_0 - e}{\sigma'_1 - \sigma'_0}$$

It is defined as the decrease in void ratio per unit increasing effective stress " $d\sigma'$ ".

UNITS :-  $m^2 / kN$  (or)  $mm^2 / N$ .

co-efficient of volume change ( $m_v$ ) :- It is defined as volume change per unit increasing effective stress.

$$m_v = - \frac{\Delta V}{V_0} \times \frac{1}{\Delta \sigma'} \quad \text{(or)} \quad m_v = \frac{a_v}{1 + e_0}$$

$$\Delta H = m_v H \Delta \sigma'$$

Compression Index ( $C_c$ ) :- It is defined as "It is equal to slope of the linear portion of the void ratio  $v_s$   $\log \sigma'$  plot".

$$\therefore C_c = \frac{-\Delta e}{\log_{10} \left( \frac{\sigma'_1}{\sigma'_0} \right)} \quad \text{(where } \sigma'_1 = \sigma'_0 + \Delta \sigma')$$

$$C_c = \frac{-\Delta e}{\log_{10} \left( \frac{\sigma'_0 + \Delta \sigma'}{\sigma'_0} \right)}$$

$$C_c \times \log_{10} \left( \frac{\sigma'_0 + \Delta \sigma'}{\sigma'_0} \right) = \frac{\Delta H}{H} \times (1 + e_0)$$

$$\Delta H = \frac{H \times C_c}{1 + e_0} \log_{10} \left( \frac{\sigma'_0 + \Delta \sigma'}{\sigma'_0} \right)$$

Q. A soft sample of height 25mm which has been subject to a consolidation test. has an area of 50 cm<sup>2</sup>. dry weight of the soil is 190.2 grams,  $q = 2.67$  then find height of the soil will change & void ratio?

Sol:-

Area (A) = 50 cm<sup>2</sup>

H = 25 mm.

$q = 2.67$

$W_d = 190.2$  grams

$H_c = ?$        $\Delta e = ?$



$$\therefore H_s = \frac{W_d}{G A \gamma_w} = \frac{190.2}{267 \times 50 \times 1} = 1.42 \text{ cm} = 14.2 \text{ mm}$$

$$\therefore \Delta e = \frac{H - H_s}{H_s} = \frac{25 - 14.2}{14.2} = 0.76 \%$$

\* In a consolidation test the following results can be opted. The load ~~was~~ changed from  $50 \text{ kN/m}^2$  to  $100 \text{ kN/m}^2$ , void ratio is changed from  $0.7$  to  $0.65$  then determine co-efficient of volume change, co-efficient of compressibility? and compression index?

Sol:-  $e_0 = 0.72$ ,  $e = 0.65$

$\sigma_1' = 100 \text{ kN/m}^2$ ,  $\sigma_0' = 50 \text{ kN/m}^2$

co-efficient of compressibility

$$a_v = \frac{e_0 - e}{\sigma_1' - \sigma_0'} = \frac{0.72 - 0.65}{100 - 50}$$

$$\therefore a_v = 1.4 \times 10^{-3} \text{ m}^2/\text{kN}$$

co-efficient of index ( $C_c$ ) =  $\frac{-\Delta e}{\log_{10}(\frac{\sigma_1'}{\sigma_0'})}$

$$= \frac{0.07}{\log_{10}(\frac{100}{50})}$$

co-efficient of volume change

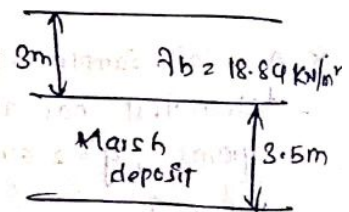
$$v = \frac{a_v}{1 + e_0} = \frac{1.4 \times 10^{-3}}{1 + 0.72}$$

\* A Sample having bulk density of  $18.84 \text{ kN/m}^3$ , is to be placed on a compressible saturated marsh deposit, of thickness  $3.5 \text{ m}$  the height of the sample is  $3 \text{ m}$ , if volume of compressibility of deposit is  $7 \times 10^{-4} \text{ m}^2/\text{kN}$ .

Sol:- Given data is

bulk density ( $\gamma_b$ ) =  $18.84 \text{ kN/m}^3$

vol of compressibility  $M_v = 7 \times 10^{-4} \text{ m}^2/\text{kN}$



$$\therefore \Delta H = M_v \times H \times \Delta \sigma$$

W.K.T  $\Delta \sigma = \gamma_b \times H = 18.84 \times 3 = 56.52 \text{ kN/m}^2$

$$\Delta H = H \times \Delta \sigma = 7 \times 10^{-4} \times 3 \times 56.52$$

$$\therefore \Delta H = 0.11 \text{ m}$$

② A clay stratum of 5m thick, has initial void ratio of 1.5 and effective over load pressure of  $120 \text{ kN/m}^2$ , when sample is subjected to increase of pressure,  $120 \text{ kN/m}^2$ , the void ratio reduces to 1.44. determine the final settlement of the stratum.

Sol:- Given data is

$$H = 5 \text{ m}$$

$$e_0 = 1.5, e = 1.44$$

$$\sigma_0' = 120 \text{ kN/m}^2$$

$$\text{Increase of pressure } \sigma_1' = \sigma_0' + \Delta\sigma'$$

$$\sigma_1' = 120 + 120$$

$$\sigma_1' = 240 \text{ kN/m}^2$$

$$\therefore \Delta H = m_v \times H \times \Delta\sigma$$

$$\therefore m_v = \frac{\alpha_v}{1+e_0} = \frac{\Delta e}{\Delta\sigma} \times \frac{1}{1+e_0}$$

$$m_v = \frac{1.5 - 1.44}{120} \times \frac{1}{1 + 1.5} = 2 \times 10^{-4} \text{ m}^2/\text{kN}$$

$$\therefore \Delta H = 2 \times 10^{-4} \times 5 \times 120 = 0.12 \text{ m}$$

\* A saturated soil has compression index  $C_c = 0.27$ , its void ratio at a stress of  $125 \text{ kN/m}^2$  is 2.04 and its permeability is  $3.5 \times 10^{-8} \text{ cm/sec}$ . compute the change in void ratio if effective stress is increased  $187.5 \text{ kN/m}^2$ . The settlement of the soil if its thickness is 5cm.

Sol:-

Given data is

$$\text{Compression index } (C_c) = 0.27$$

$$e_0 = 2.04$$

$$k = 3.5 \times 10^{-8} \text{ cm/sec}$$

$$\sigma_0' = 125 \text{ kN/m}^2$$

$$e = ?$$

$$\sigma_1' = 187.5 \text{ kN/m}^2$$

$$\Delta H = ?$$

$$\text{and } H = 5 \text{ cm}$$

$$C_c = \frac{-\Delta e}{\log_{10} \left( \frac{\sigma_1'}{\sigma_0'} \right)}$$

$$0.27 = \frac{e_0 - e}{\log_{10} \left( \frac{187.5}{125} \right)}$$

$$0.27 \times \left( \log_{10} \left[ \frac{187.5}{125} \right] \right) = 2.04 - e$$



$$\Delta e = 1.93$$

W. Know that

$$\frac{\Delta H}{H} = \frac{\Delta e}{1 + e_0}$$

$$\Delta H = H \times \frac{\Delta e}{1 + e_0}$$

$$\Delta H = 5 \times \frac{2.04 - 1.93}{1 + 2.04} = 0.18 \text{ cm}$$

\* The Laboratory consolidation data for undisturbed clay sample are as follows.  $e_1 = 1$ ,  $\sigma_1 = 85 \text{ kN/m}^2$ ,  $e_2 = 0.8$ ,  $\sigma_2 = 465 \text{ kN/m}^2$ ,  $e_3 = ?$ ,  $\sigma_3 = 600 \text{ kN/m}^2$

Sol:- for the same soil sample  $C_c$  (Compression index) is same.

$$e_1 = 1 \text{ then } \sigma_1 = 85 \text{ kN/m}^2$$

$$e_2 = 0.8 \text{ then } \sigma_2 = 465 \text{ kN/m}^2$$

$$e_3 = ? \text{ then } \sigma_3 = 600 \text{ kN/m}^2$$

$$\therefore C_c = \frac{-\Delta e}{\log_{10} \left( \frac{\sigma_1'}{\sigma_0'} \right)} = \frac{e_1 - e_2}{\log_{10} \left( \frac{\sigma_2'}{\sigma_1'} \right)} = \frac{1 - 0.8}{\log_{10} \left( \frac{465}{85} \right)}$$

$$\therefore C_c = 0.27$$

$$\text{then } C_c = \frac{\Delta e}{\log_{10} \left( \frac{\sigma_3'}{\sigma_2'} \right)} = \frac{e_2 - e_3}{\log_{10} \left( \frac{600}{85} \right)}$$

$$0.27 = \frac{0.8 - e_3}{\log_{10} \left( \frac{600}{85} \right)}$$

$$\therefore e_3 = 0.77$$

\* Calculate the final settlement of the clay layer shown in the figure due an increase in pressure of  $30 \text{ kN/m}^2$  at mid height of the clay layer. also calculate the settlement when water table rises to the ground surface.

Sol:- Given data is

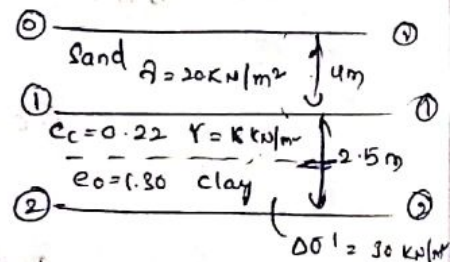
Case (i) No water table  
effective stress =  $\gamma_s K H + \gamma_c \times H$

$$\begin{aligned} \sigma_0 &= 4 \times 20 + 18 \times 1.5 \\ &= 102.5 \text{ kN/m}^2 \end{aligned}$$

$$u = 0$$

$$\sigma_1 = \sigma - u$$

$$\therefore \sigma_1 = 102.5 \text{ kN/m}^2$$

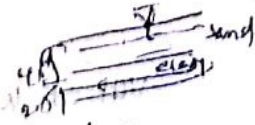


$$\Delta H = \frac{H \times C_c}{1 + e_0} \times \log_{10} \left( \frac{\sigma_1'}{\sigma_0'} \right) \quad (\sigma_1 = \sigma_0' + \Delta \sigma_1)$$

$$\Delta H = \frac{2.5 \times 0.22}{1 + 1.2} \times \log_{10} \left( \frac{102.4 + 90}{102.4} \right)$$

$$\Delta H = 0.02 \text{ m}$$

Problem :- Water table in ground



$$\sigma = \gamma H = 20 \times 4 + 18 \times \frac{2.5}{2} = 102.4 \text{ kN/m}^2$$

$$u = \gamma_w H = 10 \times (4 + 1.25) = 52.5 \text{ kN/m}^2$$

$$\sigma' = \sigma - u = 102.4 - 52.5 = 49.9 \text{ kN/m}^2$$

$$\Delta H = \frac{H \times C_c}{1 + e_0} \times \log_{10} \left( \frac{\sigma_1'}{\sigma_0'} \right)$$

$$\Delta H = \frac{2.5 \times 0.22}{1 + 1.2} \times \log_{10} \left( \frac{49.9 + 90}{49.9} \right)$$

$$\Delta H = 0.01 \text{ m}$$

### Terzaghi's one-dimensional consolidation :-

#### Assumptions :-

- \* Soil is homogenous and fully saturated
- \* Soil and water are incompressible
- \* Deformation is due to decrease in vol. of soil
- \* permeability is constant.
- \* Load is applied in a direction  $\rightarrow$  Deformation in a direction

$$u = \gamma_w H \quad \rightarrow \quad \Delta u = \frac{u}{\gamma_w}$$

$$i = \frac{u / \gamma_w}{z}$$

$$i = \frac{u}{\gamma_w z} \quad \rightarrow \quad i = \frac{1}{\gamma_w} \times \frac{\partial u}{\partial z}$$

\* Darcy's Law is applicable throughout the consolidation

for darcy's Law  $v \propto i$

$$v = k i$$

$$v = k \times \frac{1}{\gamma_w} \times \frac{\partial u}{\partial z}$$

$$\frac{\partial v}{\partial z} = \frac{k}{\gamma_w} \times \frac{\partial^2 u}{\partial z^2}$$



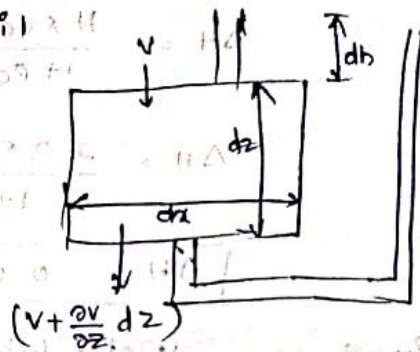


Quantity of water entering the soil is

$$Q_{in} = v \cdot A$$

$$Q_{in} = v \cdot dx \cdot dy$$

$$Q_{out} = \left( v + \frac{\partial v}{\partial z} dz \right) dx \cdot dy$$



Vol of Water squeezed out

$$\Delta Q = Q_{out} - Q_{in}$$

$$= \left( v + \frac{\partial v}{\partial z} dz \right) dx \cdot dy - v dx \cdot dy$$

$$= v dx \cdot dy + \frac{\partial v}{\partial z} dx \cdot dy \cdot dz - v dx \cdot dy = \frac{\partial v}{\partial z} dx \cdot dy \cdot dz$$

$$m_v = - \frac{\Delta v}{v_0} \approx \frac{1}{\Delta \sigma'}$$

$$\Delta v = - m_v \Delta \sigma' \cdot v_0$$

$$v_0 = \text{Initial volume} = dx \cdot dy \cdot dz$$

$$\Delta v = - m_v dx \cdot dy \cdot dz \cdot \Delta \sigma'$$

Decrease in the vol of soil per unit time

$$\frac{\partial \Delta v}{\partial t} = - m_v dx \cdot dy \cdot dz \cdot \frac{\partial \Delta \sigma'}{\partial t}$$

⇒ Vol of Water Squeezed out Unit time

= Decrease in the Vol of Soil per Unit time

$$\frac{\partial v}{\partial z} dx \cdot dy \cdot dz = - m_v dx \cdot dy \cdot dz \cdot \frac{\partial \Delta \sigma'}{\partial t}$$

$$\frac{k}{\gamma_w} \times \frac{\partial^2 u}{\partial z^2} = - m_v \frac{\partial \Delta \sigma'}{\partial t}$$

$$\text{Total stress } (\Delta \sigma) = \Delta \sigma' + u$$

Differentiate on both sides with respect to time

$$\text{Constant } \frac{\partial \Delta \sigma}{\partial t} = \frac{\partial \Delta \sigma'}{\partial t} + \frac{\partial u}{\partial t}$$

$$\frac{\partial \Delta \sigma'}{\partial t} = - \frac{\partial u}{\partial t}$$

$$\frac{k}{\gamma_w} \times \frac{\partial^2 u}{\partial z^2} = - m_v \cdot \frac{\partial u}{\partial t}$$

$$\frac{k}{\gamma_w m_v} \times \frac{\partial^2 u}{\partial z^2} = \frac{\partial u}{\partial t}$$

$$c_v = \frac{\partial^2 u}{\partial z^2} = \frac{\partial u}{\partial t}$$

$$c_v = \frac{k}{\gamma_w m_v}$$

$c_v$  Co-efficient of consolidation